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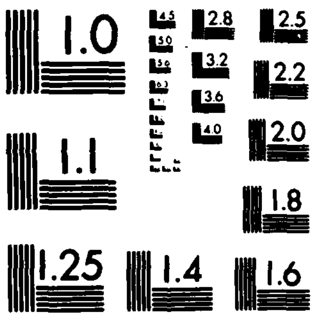
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M. Slemrod spent one half of the Spring 1983 semester at the Institute for Mathematics and its Applications. During that time he interacted with colleagues, engaged in research, and gave two public lectures at the Institute: (i) Chaos in Phase Transitions, (ii) Dynamics of Phase Transitions. Tye main thrust of Slemrod's research was in two areas, specifically: (i) Deterministic chaos in materials exhibiting phase transitions, and cont. (over)

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(ii) Admissibility criteria for weak solutions of the non-hyperbolic conversation laws which describe dynamic phase transitions.

AFOSR-TR- 83-0929

Final Progress Report

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1 June 1983

AFOSR-82-0246

0. Introduction

the author ~~M. Slemrod~~ spent one half of the Spring 1983 semester at the Institute for Mathematics and its Applications. During that time he interacted with colleagues, engaged in research, and gave two public lectures at the Institute:

~~(i)~~ Chaos in Phase Transitions, and

~~(ii)~~ Dynamics of Phase Transitions.

The main thrust of his ~~Slemrod's~~ research was in two areas, specifically:

~~(i)~~ Deterministic chaos in materials exhibiting phase transitions, and

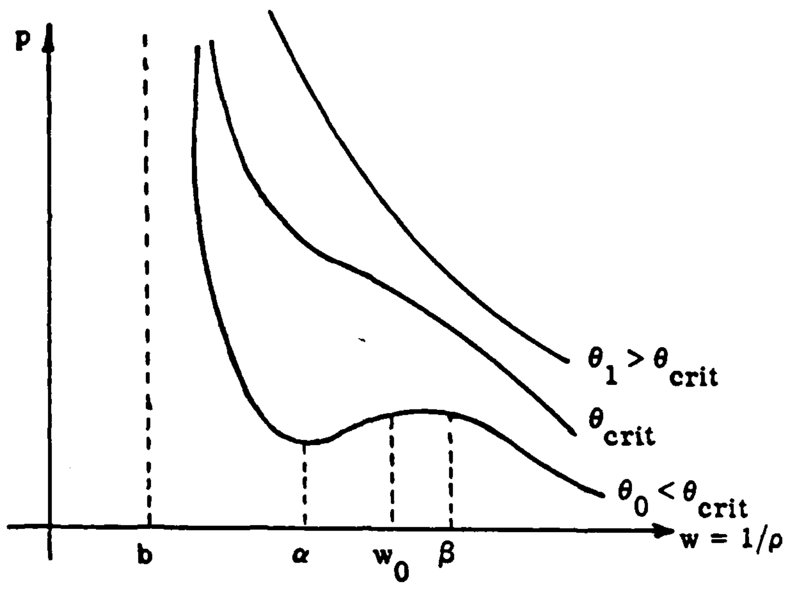
~~(ii)~~ Admissibility criteria for weak solutions of the non-hyperbolic conservation laws which describe dynamic phase transitions.

A detailed description of this research is given below.

1. Chaos in Materials Exhibiting Phase Transitions

In recent work Holmes and Marsden (1981) have employed the Mel'nikov (1963) technique to establish the existence of deterministic chaos in periodically forced evolution equations. In this work Slemrod (in collaboration with J.E. Marsden of the University of California, Berkeley) combined this program by analyzing two problems arising from the van der Waals theory of phase transitions [see van der Waals (1893, 1979)].

The first problem considered was dynamic spinoidal decomposition. Assume we have a van der Waals fluid with isotherms as shown in Figure 1.



Here p denotes the pressure given by the van der Waals constitutive relation

$$p(w,\theta) = \frac{R\theta}{w-b} - \frac{a}{w^2} , \qquad (1.1)$$

R, b, a are positive constants, ρ is the density, $w = 1/\rho$ the specific volume, and θ is the absolute temperature. Figure 1 sketches the van der Waals isotherms

for θ above, equal to, and below the critical temperature $\theta_{crit} = 8a/27bR$. [A good reference is Fermi (1936).] For $\theta_0 < \theta_{crit}$, $p(w,\theta)$ has the following features:

(i) $p_w(w,\theta_0) < 0$ on $(b,\alpha) \cup (\beta,\infty)$,

(ii) $p_w(w,\theta_0) > 0$ on (α,β) ,

(iii) $p_w(\alpha,\theta_0) = p_w(\beta,\theta_0) = 0$.

The domain (b,α) corresponds to the fluid being liquid; the domain (β,∞) corresponds to the fluid being vapor; the domain (α,β) is the unstable region and is referred to as the spinoidal. Also the point w_0 is where $p_{ww} = 0$ on the graph of the θ_0 isotherm (w_0 is the zero of $R\theta_0 w^4 + a(w-b)^3 = 0$).

The fluid flow is thought of as taking place along the x-axis in a tube of unit cross section of fixed volume. If $\theta_1 > \theta_{crit}$ the state $w = w_0$ will describe a stable homogeneous configuration for the hydrodynamic equations [see Felderhof (1970), Slemrod (1982)]. We then "instaneously" quench the fluid by reducing the temperature to $\theta_0 < \theta_{crit}$. The homogeneous state w_0 will now be in the unstable spinoidal region of the θ_0 isotherm. We studied the effect of a small <u>periodic in time</u> fluctuation of the absolute temperature about θ_0. Specifically we showed how the loss of stability of w_0 is accompanied by deterministic chaos in that there will be a Poincaré-Birkhoff-Smale horseshoe in the dynamics.

The second problem considered is the effect of a small thermal perturbation, <u>periodic in space</u>, of the form $\theta(x) = \theta_0 + \epsilon \cos qx$, ϵ small, on the equilibrium configuration of an infinite tube of liquid under given applied load. In this case we show there are solutions with the features of both metastable and co-existing phases that exhibit spatial chaos.

It seems interesting to note that the above equilibrium result has features in common with experimental observations presented by Dr. L. Zapas of NBS at the IMA Workshop on Orienting Polymers, March 21-25, 1983.

The results of this research will appear as an IMA report and will be submitted for publication.

2. Admissibility Criteria for Weak Solutions of Conservation Laws

In recent years an ever-increasing number of people have taken up Gelfand's (1963) program of parabolic regularization of quasilinear conservation laws. While this work has usually been motivated by the search for reasonable admissibility criteria for weak solutions of the conservation laws, it has also provided a technique for proving existence of solutions as well. Specifically in this regard we note the work of Oleinik (1957) for a scalar conservation law and the recent remarkable results of DiPerna (1983) for a two-dimensional system of conservation laws. In light of this work Slemrod attempted to give (where possible) a physical motivation to the Gelfand program with respect to the equations of compressible fluid flow.

Consider a system of n conservation laws written in vector form

$$F(U)_t + G(U)_x = 0 \tag{2.1}$$

where $U(x,t)$ is an n-vector, $-\infty < x < \infty$, and $t > 0$. As is well known, the initial value problem (2.1) and

$$U(x,0) = U_0(x) \tag{2.2}$$

does not in general possess global smooth solutions if (2.1) is nonlinear, no matter how smooth U_0. Hence we are led to search for weak solutions of (2.1). Unfortunately, there is a serious price to pay for enlarging the solution class, namely loss of uniqueness of solutions.

The Gelfand program of parabolic regularization attempts to give a way to pick out the physically relevant weak solutions of (2.1) and thereby, hopefully, recover uniqueness of weak solutions of (2.1). Specifically Gelfand suggested that the physically admissible solutions of (2.1) should be limits of solution U^ϵ of the "artifically viscous" equations

$$F(U^\epsilon)_t + G(U^\epsilon)_x = \epsilon D(U^\epsilon_x)_x \tag{2.3}$$

where $D: \mathbb{R}^n \longrightarrow \mathbb{R}^n$, $\epsilon > 0$ a small parameter. For example, the equations of viscous isothermal compressible fluid flow in Lagrange coordinates are

$$u_t^\epsilon + p(w^\epsilon)_x = \epsilon u_{xx}^\epsilon$$
$$w_t^\epsilon - u_x^\epsilon = 0 \tag{2.4}$$

so that $U^\epsilon = \begin{pmatrix} u^\epsilon \\ w^\epsilon \end{pmatrix}$ and $D = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$.

Unfortunately D in this example is singular, whereas the recent results of DiPerna on existence of weak solutions of (2.1) as limits of (2.3) require D nonsingular.

Slemrod in his research attempted to give a physical justification for nonsingular D matrices based on the van der Waals-Korteweg gradient theory of stress. Using this idea Slemrod was able to apply DiPerna's technique to prove the existence of weak solutions of (2.4, $\epsilon = 0$) for all $t \geq 0$ when p is described by the θ_{crit} isotherm of Fig. 1.

This work will appear as an IMA report and will be submitted for publication.

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